A large scale integer linear programming to the daily fleet assignment problem: a case study in Turkey

Yavuz Ozdemir a *, Huseyin Basligil a, Baglan Sarsenov a

a Industrial Engineering Department, Yildiz Technical University, Barbaros Bulvari, Yildiz, Istanbul, 34349, Turkey

Abstract

Since poor fleet assignment can cause a great increase in costs for airline companies, a solution of the type ‘right fleet for the right flight’ would be very useful. In this paper, a fleet assignment model is set up using the data of the largest Airline Company in Turkey, Turkish Airlines. The aim of this model is to assign most appropriate fleet type to flights while minimizing the cost and determine optimal number of aircraft grounded overnight at each airport. The contributions of this study to Turkish Airlines will be calculated financially and the cost minimizations can be shown with the given data.

1. Introduction and Literature Review

The transportation sector, providing transportation of people or goods from one destination to another started to become more and more important in our globalizing world, which becomes a smaller village every single day. Airway transportation is one of the transportation types that succeeded in putting itself forward in the beginning of the 20th century among transportation types. In airline operations, schedule development involves many steps, including schedule design, fleet assignment, aircraft routing, and crew pairing (Etschmaier & Mathaisel, 1985).

The problem of fleet assignment is one of the hardest and most comprehensive problems faced in airline planning. Assigning fleet types to flight legs effectively is crucial in airline planning because the objective is to minimize cost to the airline. The aim of fleet assignment is to match most appropriate fleet type to flights while minimizing the cost. It should be noted that this planning concerns only fleet type, not a particular aircraft.

The fleet assignment model is usually formulated for a typical day. And attempts to solve the fleet assignment problem have used various optimization methods. Belanger et al. (2006a) presented a mixed-integer linear programming formulation for the fleet assignment problem with homogeneity and showed that it is possible to produce very good quality solutions using a heuristic mixed-integer programming approach. Abara (1989) formulated the solution to the fleet assignment problem as an integer linear programming model, permitting assignment of two or more fleets to a flight schedule simultaneously.

Belanger et al. (2006b) proposed a model for the periodic fleet assignment problem with time windows in which departure times are also determined. A weekly fleet assignment model is presented by Kliwijer and Tschöke (2000).
They use a simulated annealing (SP) approach to deal with higher complexity. Chung and Chung (2002) attempted to solve the fleet assignment problem using genetic algorithms.

In fleet assignment, profit is maximized by minimizing two types of costs: operational and spill costs (Subramanian et al., 1994). Operational costs are those for flying the flight leg with the assigned aircraft type and usually include such things as fuel and landing fees. Spill costs represent lost opportunity costs that arise if passenger demand exceeds the aircraft capacity and, thus, potential revenue is lost (Barnhart et al., 2003).

Bazargan introduced operating costs in the fleet assignment model, with passenger-spill costs, recapture rate, flight cover, etc. An optimum solution is found in Bazargan’s study (2004).

In this paper, we discuss the fleet assignment problem, one of the most important problems with which airline companies must deal. We use the same model engaged by Bazargan but with real data of the largest airline company in Turkey, Turkish Airlines. The main contribution of this paper to the literature will be the ability it provides to see cost reduction, and to optimize fleets by using real case data. This is the first paper that discusses fleet assignment problem for Turkey with too large problem dimensions (hub, spokes, and number of fleets), which is so close to daily domestic airline fleet assignment for Turkish Airlines. This paper is the extension of (Ozdemir et al., 2012).

In our case, the hub selected is Istanbul Ataturk Airport and the spokes are Ankara, Antalya, Adana, Gaziantep, Izmir, Hatay, Erzurum, Samsun, Van, Konya, Kayseri, Trabzon and Diyarbakir as seen on Figure 1.

An easier solution to this problem is ensured using optimization software, Lindo 6.1. A fleet assignment model was set up using the data of Turkish Airlines, with linear integer programming. Firstly, the objective function for the Turkish Airlines model was set up with selecting our binary and integer decision variables. Then operation costs and passenger spill costs were calculated for each fleet type. These values were calculated for each flight in the Turkish Airlines flight schedule according to expected demand and standard deviation for flights considering recapture rate. After the objective function was determined, the fleet assignment model was set up with respect to flight cover, aircraft balance and fleet size constraints. After all these calculations were made, our fleet assignment model was constructed.

The paper is organized as follows. In Section 2, we present the general mathematical model for the fleet assignment problem. In Section 3, we set up the objective function of the mathematical model for the fleet assignment problem and the three main sets of constraints - flight cover, aircraft balance, fleet size - are discussed in the fleet assignment model. Finally, we make an application for a fleet assignment model with the Turkish Airlines case. In Section 4, we present some conclusions.

2. Fleet Assignment Model (FAM)

We now present the general mathematical model for the fleet assignment problem. The following model (Bazargan, 2004), referred to as the basic fleet assignment model (FAM), is a simplified version of FAM proposed by Hane et al. (1995).

Sets
- F = Set of flights
- K = Set of fleet types
- C = Set of last-nodes, representing all nodes with aircraft grounded overnight at an airport in the network
- M = Number of nodes in the network
Index
\( i \) = Flight Index
\( j \) = Index for fleet
\( k \) = Index for nodes

Parameters
\( C_{i,j} \) = Cost of assigning fleet type \( j \) to flight \( i \)
\( N_j \) = Number of available aircraft in fleet type \( j \)
\( S_{i,k} \) = +1 if flight \( i \) is an arrival at node \( k \), -1 if flight \( i \) is a departure from node \( k \)

Decision Variables
\( x_{i,j} = \begin{cases} 
1 & \text{if flight is assigned to fleet-type } j, \text{ 0 otherwise} 
\end{cases} \)

\( G_{k,j} = \) integer decision variable representing number of aircraft of fleet-type on ground at node \( k \)

The integer linear programming model is as follows:

\[
\text{min} \sum_{j \in K} \sum_{i \in F} C_{i,j} x_{i,j} \\
\text{Subject to} \\
\sum_{j \in K} x_{i,j} = 1 \quad \forall i \in F \\
G_{k,j} + \sum_{i \in F} S_{i,k} x_{i,j} = G_{k,j} \quad \forall k \in M \text{ and } \forall j \in K \\
\sum_{i \in F} G_{k,j} \leq N_j \quad \forall j \in K \\
x_{i,j} \in \{0,1\} \quad \forall i \in F \text{ and } \forall j \in K \\
G_{k,j} \in Z^+ \quad \forall k \in M \text{ and } \forall j \in K
\]

In the above model, the objective function in (1) seeks to minimize the total cost of assigning the various fleet types to all the flights in the schedule. Constraints (2) are the flight-cover constraints to ensure that each flight is flown by one type of fleet. Constraints (3) are the aircraft balance constraints. Set of constraint (4) represents the fleet size. The number of aircraft in fleet type \( j \), should not exceed the available number of aircraft in that fleet (\( N_j \)). Constraints (5) and (6) represent the binary and integer status of the decision variables. \( Z^+ \) is the set of positive integer numbers (Bazargan, 2004).

3. An Application in Turkish Airlines

For the fleet assignment problem with linear integer programming that we studied in this paper, there are 4 aircraft for A319 fleet type, 25 aircraft for A320 fleet type, 21 aircraft for A321 fleet type, 14 aircraft for B737 fleet type and 52 aircraft for B738.

Some of the flight schedule route (all of 144 flights per day), is presented in Table 1. It is assumed that demand for each flight is normally distributed with given means and standard deviations as seen on Table 1. Also Table 1 presents the demand distribution for each flight as well as distances between cities. The mean of demand and standard deviation are taken from the historical data for 2 years.
Table 1. Flight Schedule, destination in km, demand means and standard deviations for Turkish Airlines network

<table>
<thead>
<tr>
<th>Flight no.</th>
<th>Origin</th>
<th>Departure time</th>
<th>Destination</th>
<th>Arrival time</th>
<th>Demand</th>
<th>Standard deviation</th>
<th>Distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TK2451</td>
<td>Adana</td>
<td>03:35</td>
<td>Istanbul</td>
<td>05:15</td>
<td>146</td>
<td>24</td>
<td>695</td>
</tr>
<tr>
<td>TK2237</td>
<td>Gaziantep</td>
<td>03:45</td>
<td>Istanbul</td>
<td>05:30</td>
<td>139</td>
<td>21</td>
<td>851</td>
</tr>
<tr>
<td>TK2403</td>
<td>Antalya</td>
<td>04:00</td>
<td>Istanbul</td>
<td>05:15</td>
<td>140</td>
<td>21</td>
<td>475</td>
</tr>
<tr>
<td>TK2305</td>
<td>Izmir</td>
<td>04:20</td>
<td>Istanbul</td>
<td>05:25</td>
<td>137</td>
<td>20</td>
<td>402</td>
</tr>
<tr>
<td>TK2105</td>
<td>Ankara</td>
<td>04:25</td>
<td>Istanbul</td>
<td>05:30</td>
<td>140</td>
<td>23</td>
<td>402</td>
</tr>
<tr>
<td>TK2257</td>
<td>Hatay</td>
<td>05:00</td>
<td>Istanbul</td>
<td>06:50</td>
<td>163</td>
<td>33</td>
<td>784</td>
</tr>
<tr>
<td>TK2116</td>
<td>Istanbul</td>
<td>06:00</td>
<td>Ankara</td>
<td>07:05</td>
<td>168</td>
<td>33</td>
<td>402</td>
</tr>
</tbody>
</table>

Some calculations can be done as:
- Operating costs of a flight = Cost per available seat mile (CASM) of the fleet × Distance × Number of seats on the aircraft
- Expected spill cost for a fleet = Expected number of passenger spill × Revenue per Available seat mile (RASM) × Distance
- Expected spill costs = Expected spill cost × (1-recapture rate)
- Total Cost = Operating Costs + Passenger-Spill Costs

Objective function and constraints (flight cover, aircraft balance, fleet size) are as seen on Eq. 1-6.

3.1. Solution to the fleet assignment problem

The integer linear program for fleet assignment for Turkish Airlines has 2160 (720 binary and 1440 integer) variables and 1588 constraints. By using optimization software, the solution to this problem generates a minimum daily cost of fleet assignment of $732,872. However, prices are not certain because of firm politics. Table 2 shows the number of aircraft for each fleet type staying overnight at some of the airports. Other aircrafts which are not shown in Table 2 will be located in Istanbul during the night, because of parking, nightly maintenance, and probability of requirements to other destinations. These numbers represent the right number of aircraft for each fleet type at the right airport at the right time. And Table 3 presents some of the assignment of each flight to either one of the five fleet types.

Table 2. Optimal number of aircraft grounded overnight at each airport

<table>
<thead>
<tr>
<th>Airports</th>
<th>A319 Fleet</th>
<th>A320 Fleet</th>
<th>A321 Fleet</th>
<th>B737 Fleet</th>
<th>B738 Fleet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adana</td>
<td>-</td>
<td>4</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ankara</td>
<td>-</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Antalya</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Diyarbakır</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Erzurum</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Gaziantep</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Hatay</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Istanbul</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>
4. Conclusion and Further Researches

Air transportation plays a supplementary role in our life. It represents the fastest way to ship over long distances, and people prefer air transportation for vacations, business trips, and almost any travelling needs. Consequently, airline planning has become very important.

As an extension of (Ozdemir, 2012), in this paper we set up a model for efficient fleet assignment, and studied a real-world case study for the largest Airline Company in Turkey, Turkish Airlines. This model was coded and solved with optimization software, using linear integer programming. The aim was minimizing the cost, and the benefits of this study are explained below:

- The determination of a Fleet Type Assigned to a Specific Flight. The importance of this is to minimize the assignment cost while assigning the right aircraft to the right flight. The solution to this problem generates a minimum daily cost of fleet assignment of $732,872.

- Optimizing the Number of Aircraft that will be Grounded Overnight at each Airport. To impede any negative situation while assigning fleets during a time horizon, it is very important to know at least how many aircraft must be on the ground during the night in all of the airports.

As a further research, we plan to work with a weekly or monthly schedule in the same way we studied daily assignment in this study. We also plan to define route planning and crew scheduling problems and plan to integrate them with this model. Using this model and integrating variables from other problems with it, we can develop new solution algorithms; ones that will be more appropriate for real world cases and help solve airline problems more easily and quickly.

References